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# FOREIGN TECHNOLOGY DIVISION



SUPPORTING SURFACE IN NONSTATIONARY FLOW NEAR SCREEN

by

B. N. Belousov, A. N. Lukashenko, A. N. Panchenkov





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## **EDITED TRANSLATION**

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By: B. N. Belousov, A. N. Lukashenko, A. N. Panchenkov

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PREPARED BY:

TRANSLATION DIVISION FOREIGN TECHNOLOGY DIVISION WP-AFB, OHIO.

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Block	Italic	Transliteration	Block	Italic	Transliteration
A a	Aa	А, а	Рр	PP	R, r
Бб	<b>5</b> 6	B, b	Сс	Cc	S, s
Вв	B •	V, v	Тт	T m	T, t
Гг	Γ .	G, g	Уу	Уу	U, u
Дд	Д д	D, d	Фф	Ø Ø	F, f
Еe	E .	Ye, ye; E, e*	X ×	X x	Kh, kh
Ж ж	ж ж	Zh, zh	Цц	4	Ts, ts
3 э	3 ,	Z, z	4 4	4 4	Ch, ch
Ии	н и	I, i	Шш	Шш	Sh, sh
Йй	A a	У, у	Щщ	Щщ	Sheh, sheh
Н н	KK	K, k	Ъъ	2 1	tt
Лл	ЛА	L, 1	Ыы	ы ы	Y, у
ММ	M M	M, m	Ьь	<b>b</b> •	•
Нн	H N	N, n	Ээ	9 ,	E, e
0 0	0 0	0, 0	Юю	10 n	Yu, yu
Пп	Пп	P, p	Яя	Яя	Ya, ya

<sup>\*</sup>ye initially, after vowels, and after ь, ь elsewhere. When written as ë in Russian, transliterate as yë or ë. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

#### GREEK ALPHABET

Alpha	Α	α	•	Nu	N	ν	
Beta	В	β		Xi	Ξ	ξ	
Gamma	Γ	Υ		Omicron	0	0	
Delta	Δ	δ		Pi	П	π	
Epsilon	E	ε	ŧ	Rho	P	ρ	•
Zeta	Z	ζ		Sigma	Σ	σ	ς
Eta	Н	η		Tau	T	τ	
Theta	Θ	θ	\$	Upsilon	T	υ	
Iota	I	ι		Phi	Φ	φ	φ
Kappa	K	n	K	Chi	X	χ	
Lambda	Λ	λ		Psi	Ψ	Ψ	
Mu	М	μ		Omega	Ω	ω	

### RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russ	sian	English	
sin		sin	
cos		cos	
tg		tan	
ctg		cot	
sec		sec	
cose	ec	csc	
sh		sinh	
ch		cosh	
th		tanh	
cth		coth	
sch		sech	
csch	n	csch	
arc	sin	sin-l	
arc	cos	cos-l	
arc	tg	tan-1	
arc	ctg	cot-1	
arc	sec	sec-1	
arc	cosec	csc-l	
arc	sh	sinh <sup>-1</sup>	
arc	ch	cosh <sup>-1</sup>	
arc	th	tanh-1	
arc	cth	coth <sup>-1</sup>	
arc	sch	sech-1	
arc	csch	csch-1	
rot		curl	
lg		log	

## log lg

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#### SUPPORTING SURFACE IN NONSTATIONARY FLOW NEAR SCREEN

B. N. Belousov, A. N. Lukashenko, A. N. Panchenkov

At the present time there exist a number of important studies on the theory of a supporting surface in a nonstationary flow [1, 2, 3] with different approximate methods of calculating the wing. As we know, the problem of a thin supporting surface of arbitrary length is reduced to two-dimensional integral equations for which there are no closed solutions.

One of the most important aspects is that of studying two-dimensional integral equations in one-dimensional approximations and obtaining on this basis final results for the hydroaerodynamic characteristics of a wing in a nonstationary flow.

Whereas for a wing in a stationary flow Prandtl's theory of the supporting line defines the Prandtl equation in terms of the one-dimensional approximation, for a wing in a nonstationary flow the complexity of the physical phenomenon does not provide a unified approximation method. Thus, there are a large number of approaches (up to 20, as noted by R. A. Weisplinghoff, H. Ashley, and R. A. Halfman [1]) which give different one-dimensional equations.

Developed in the works of one of the authors [6, 7, 8, 9] is a method of integral operators for the theory of a lifting surface in a nonstationary flow.

The general solution to the problem is represented in the form of three components. Two components are found from equations which correspond in form to the equations of the theory of a wing in a nonstationary flow.

Also examined in [8] is a method of constructing the Prandtl equation based on the example of a wing in an unlimited fluid flow. A significant advantage of this method is that the Prandtl equation is constructed for the parametric constant of a singular solution. This has made it possible to more deeply analyze the problem and critically examine the existing theories of Reisner [1], Kusner [2], and others.

In the present article a method of integral operators is applied to the problem of a supporting surface near a screen. Obtained are two-dimensional integral equations and a one-dimensional intego-differential equation for the parametric constant. It should be mentioned that the downwash is associated only with the singular solution corresponding to a singularity in the leading edge of the wing. Transfer of this result to the theory of stationary motion indicates that the classical theory of a lifting line is strictly applicable only to a plane lifting surface.

If the lifting surface is not planar, then in the expression for the coefficient of the lifting force of the wing there appear terms on the order of  $1/\lambda$  which are not considered by the classical theory. From the physical standpoint they determine the effect of aspect ratio on the angle of zero lift of the wing.

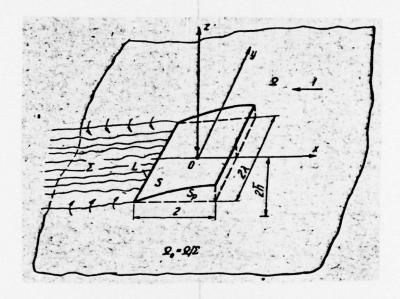
The present study adheres very closely to studies [6, 7, 8, 9], for which reason we have attempted to very briefly introduce the general results from these studies in the necessary cases and discuss new results in more detail.

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l. Let us assume that a thin lifting surface S moves at velocity  $V_0$  near a solid screen. In additional to the main forward motion it performs harmonic oscillations at frequency  $\omega$ .

Assuming that the disturbances introduced into the flow by the lifting surface are small, in coordinate system oxyz moving at velocity  $V_0$  (see the figure) let us use a known method to linearize the problem [4, 5].



In the case of harmonic oscillations for hydrodynamic potentials of velocities  $\phi$  and accelerations  $\theta$  and their derivatives we have

$$\Theta_{x_i}^n(g; l) = \overline{\Theta}_{x_i}^n(g) e^{i\omega l}, 
\varphi_{x_i}^n(g; l) = \varphi_{x_i}^n(g) e^{i\omega l} g \in \Omega_0.$$
(1.1)

The index n denotes the derivative of the n-order of functions  $\theta$  and  $\phi$  with respect to coordinate  $x_i(x_1 = x; x_2 = y; x_3 = z)$ .

In equations (1.1) and henceforth we shall only study the real parts of the proper expressions, although we shall drop Re

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for simplicity. Relationships (1.1) make it possible to formulate the problem for  $\overline{\Theta}(g)$  and  $\overline{\varphi}(g)$  and exclude time from our study.

By shifting to dimensionless space  $\Omega$ , assuming that the velocity of the impinging flow equals 1 and removing the bar from the top of corresponding  $\Theta(g)$  and  $\varphi(g)$  expressions, we get the boundary problem in the space of the potential of the accelerations  $\Theta$ :

$$\Delta \Theta = 0; \quad g \in \Omega; 
\Theta_g = F(g); \quad g \in S_\rho;$$
(1.2)

(A)

$$\theta_z = 0; z = 2\bar{h}; 
\theta_+ - \theta_- = 0, g \in L; 
\theta \to 0; z \to + \infty.$$
(1.3)

Problem (A) is obtained by integral operator Ay, assigned in space  $L_1(S_p)$  with values of  $C^2\Omega$ . The required properties of operator Ay are given in [5-8].

In the studied problem operator Ay has the form of

$$A\gamma = \frac{1}{4\pi} \int_{Sp} \gamma(p) \frac{d}{dt} \left[ \frac{1}{t} + \operatorname{sign} F \frac{1}{t_1} \right] dS; \qquad (1.4)$$

here

$$r = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2};$$

$$r_1 = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z+\zeta+4\bar{h})^2};$$

sign 
$$F = \begin{cases} -1 - \text{ solid screen} \\ +1 - \text{ free surface.} \end{cases}$$

According to condition (1.2) we get the two-dimensional integral equation of the problem

$$\overline{A}_{el} = F(g), g \in S_{p}$$
 (1.5)

Let us represent the solution in the form of three components [6-8]:

$$\gamma(g) = \gamma_1(g) + \gamma_2(g) + \gamma_3(g)$$
  $g \in S_p$ ,

where  $\gamma_1$  is the solution of class  $C^1(S_p)$ , related to the presence of a break in tangent velocities during transfer of the surface  $S_p$ ;  $\gamma_2$  - solution of class  $C^1(S_p)$ , which describes inertial movements;  $\gamma_3$  - singular solution.

Analysis of equation (1.5) and the method of constructing equations for  $\gamma_i$  is given in [6].

Let us write equations for  $\gamma_1$  and  $\gamma_2$ :

$$N_{01}\overline{A}_{211} = F_1 + C, \quad g \in S_p;$$
 (1.6)  
 $N_{01}\overline{A}_{211} = -ikF_1, \quad g \in S_p.$  (1.7)

Here  $F_1 = -\frac{d}{dx} F_2(g)$ ;  $F_2(g)$  - the function which describes the shape of the surface S;  $N_{01} = -\int_0^X d\tau$ .

Constant C is found from the condition of solvability of equation (1.6) in the space of  $C^1(S_p)$  [6].

By performing calculations for  $N_{\mbox{Ol}\overline{A}_{\mbox{Z}}}\gamma$  we get [10]

$$N_{01}\overline{A}_{x7} = \frac{1}{2\pi} \int_{-1}^{+1} \int_{-1}^{+1} \overline{\gamma}(p) \left\{ \frac{1}{dy} \frac{1}{(y-\eta)} \frac{\sqrt{(x-\xi)^{3} + \lambda^{3}}(\eta)(y-\eta)^{4}}{(x-\xi)} \right\} - \sin g F \frac{(x-\xi)[(y-\eta)^{3}\lambda^{3}(\eta)(\overline{r}_{1}^{3} - \lambda^{3}(\eta)16\overline{h}^{3}(\eta)] - \lambda^{3}(\eta)16\overline{h}^{2}(\eta)(\overline{r}_{1}^{3} + \lambda^{3}(\eta)16\overline{h}^{3}(\eta))]}{\overline{r}_{1}^{3}} dS, \quad (1.8)$$

where

$$\bar{\gamma}(p) = \frac{\gamma(p)}{2\lambda(n)}$$
.

The singular solution is determined from equation

$$N\overline{A}_{z1} = F_1(g); \quad h \in S_p;$$

$$N = -e^{iks} \int_{z}^{z} e^{-ik\cdot} d\tau, \qquad (1.9)$$

k is the Strouhal chord number.

Equation (1.9) is obtained by mapping equation (1.5) in the space of the velocity potential.

2. Now let us examine the Prandtl equation for a wing and a nonstationary flow. For a wing of great aspect ratio operator  $N_{01}\overline{A}_{z}\gamma$  is approximated by operator

$$N_{01}\bar{A}_{z\gamma} = \frac{\lambda(y)}{\pi} \int_{-1}^{+1} \bar{\gamma}(\xi; y) \left[ \frac{1}{(x-\xi)} + \operatorname{sign} F \frac{(x-\xi)}{(x-\xi)^2 + 16\bar{h}^2(y)} \right] dy. \tag{2.1}$$

For a plane flow the form of the singular solution  $\gamma_3$  is known, and for a wing of great aspect ratio we use  $\gamma_3$  in the form of the solution of the planar problem, although with a constant which depends on the y-coordinate:

$$\gamma_3 = a(y) \sqrt{\frac{1+S}{1-S}}.$$

If

$$\gamma_{12}^0 \in C^1(S) \text{ and } \gamma_2 = -ik \int_{-1}^x \gamma_1 d\tau; \quad g \in S_p,$$

then we get [6]

$$N\overline{A}_{z_{12}} = N_{01}\overline{A}_{z_{11}}; \quad g \in S_{p}. \tag{2.2}$$

By examining equations (1.6), (1.7), (1.9) and property (2.2), we get

$$N\overline{A}_{2\overline{1}3} = -C - N_1\overline{A}_{2\overline{1}23}$$

Equation (1.6) can be written in the form

$$N_0 \overline{A}_{271} = F_1 + C^1, \qquad (2.3)$$

$$N_0 = -\int_0^z d\tau.$$

When  $\lambda \to \infty$  C<sup>1</sup> = C, while for greater aspect ratio we use the approximation  $C = -\frac{1}{\pi} \int_{-\frac{1}{2}}^{1} \frac{[F_1 - (N_0 - N_{01}) \, \overline{A}_2 \, \gamma_1]}{V \, 1 - x^3} \, dx. \tag{2.4}$ 

Then, by introducing operator  $p^{-1}$ :

$$p^{-1}N_{01}\overline{A}_{z}\gamma_{1} = \int_{-1}^{+1} \gamma_{1}(\xi) d\xi$$

and taking approximation (2.1) from (2.3), we get the Prandtl equation for the regular solution, which in form coincides with the Prandtl equation of the stationary problem [4]:

$$\Gamma_{1}(y) = \frac{\psi \pi}{\lambda(y)} \left\{ \alpha_{1} - \frac{1}{2\pi} \int_{-1}^{+1} \Gamma'_{1}(\eta) \left[ \frac{1}{y - \eta} - \frac{y - \eta}{(y - \eta)^{2} + 16\bar{h}^{2}} \right] d \right\}$$
 (2.5)

where

$$\Gamma_1 = \int_{-1}^{+1} \gamma_1(\xi) d\xi.$$

Function  $\gamma_{22}$  is found from equation

$$N_{g1}\bar{A}_{z\bar{1}g2} = ik\,C(y). \tag{2.6}$$

Now let us introduce a number of assumptions typical for the Prandtl problem [4], [8]:

$$N_{1}\overline{A}_{z}\gamma \approx N_{10}\overline{A}_{z}\gamma + N_{1\lambda}\overline{A}_{z}\gamma;$$

$$\gamma(p) = \gamma_{1}(\xi)\gamma_{2}(y);$$

$$N_{10}\overline{A}_{z}\gamma = 2\gamma_{2}(y)\lambda(y)N_{10}\overline{A}_{z}\gamma_{1}(\xi);$$

$$N_{1\lambda} = -e^{ikx}\int_{\xi} e^{-ik\tau}d\tau;$$

$$N_{1\lambda}\overline{A}_{z}\gamma = \frac{1}{2\pi}\left\{\int_{-1}^{+1}\gamma(\xi)\left[\frac{1}{(x-\xi)} + \operatorname{sign}F\frac{(x-\xi)}{(x-\xi)^{3} + \operatorname{loh}^{3}(y)}\right]d\xi + ike^{ikx}\int_{-1}^{+1}\gamma(\xi)\int_{\xi} e^{-ik\tau}\left[\frac{1}{(\tau-\xi)} + \operatorname{sign}F\frac{(\tau-\xi)}{(\tau-\xi)^{2} + \operatorname{loh}^{3}(y)}\right]d\tau d\xi\right\},$$

then

$$a(y) 2\lambda(y) N_{10} \overline{A}_{2} \gamma_{1}(\xi) = -C(y) - 2\lambda(y) \gamma_{222}(y) N_{10} \overline{A}_{2} \gamma_{221} - -N_{1\lambda} \overline{A}_{2} \left[ a(y) \sqrt{\frac{1+S}{1-S}} + \gamma_{22} \right]; g \in S_{p}.$$
(2.7)

For a wing in an infinite fluid both parts of equation (2.7) contain factors  $e^{ikx}$ . By multiplying equation (2.7) by  $e^{-ikx}$ , we eliminate the variable x [5, 8]. This also occurs in the general case, although the equation relative to e ikx cannot be solved in explicit form, since it is possible to calculate the integrals in  $N_{10}\overline{A}_{z}\gamma$  in a closed form. In this connection let us introduce averaged equation (2.7) according to the following rule:

$$\bar{B} = \frac{\int_{-1}^{+1} B \sqrt{\frac{1-x}{1+x}} dx}{\int_{-1}^{+1} e^{ikx} \sqrt{\frac{1-x}{1+x}} dx}.$$

For  $N_{10}^{\overline{A}} y_1$  and  $\gamma_{222}(y) N_{10}^{\overline{A}} y_{221}$  we select representations in the form of

$$\overline{N_{10}}\overline{\bar{A}}_{2}\gamma_{1} = \frac{\overline{N_{1}(k)}}{\psi_{1}(k, \bar{h})2\pi}; \qquad (2.8)$$

$$\overline{N_{10}\overline{A}_{z}\gamma_{1}} = \frac{\overline{N_{1}(k)}}{\overline{\psi_{1}(k; \overline{h})2x}}; \qquad (2.8)$$

$$\overline{\gamma_{222}(y)} \, \overline{N_{10}\overline{A}_{z}\gamma_{221}} = \frac{\gamma_{222}(y)}{2\pi\psi_{2}(k; \overline{h})2\lambda(y)\lambda_{222}^{\circ}(y)} \left[ \frac{ik}{\pi} \, \overline{N}_{2}(k) \, C(y) + C_{2}(y) \, \overline{N}_{3}(k) \right]. \qquad (2.9)$$

When  $\overline{h}$  +  $\infty$  representations (2.8) and (2.9) are transformed into the expressions found in [9]. Functions  $\overline{N}_i(k)$  are also obtained in [9].

Then,

$$a(y) 2\lambda(y) \frac{\overline{N}_{1}(k)}{\psi_{1}(k; \overline{h}) 2\pi} + \frac{\gamma_{223}(y)}{2\pi\psi_{2}(\overline{k}; \overline{h}) \gamma_{232}^{0}(y)} \left[ \frac{ik}{\pi} \overline{N}_{2}(k) C(y) + C_{3}(y) \overline{N}_{3}(k) \right] = \\ = -C(y) - N_{J\lambda} \overline{A}_{2} \left[ a(y) \sqrt{\frac{1+S}{1-S}} + \gamma_{23} \right].$$
(2.10)

With the introduction of a new variable 
$$a_{\lambda}(y) = \int_{-1}^{+1} [a(y)\sqrt{\frac{1+S}{1-S}} + \gamma_{221}(y; S)] dy$$
 equation (2.8) is transformed into

 $a_{\lambda}(y) = \frac{\pi\psi_{1}(k; \bar{h})}{2\lambda(y)} \left\{ a_{0} + \frac{\pi L a_{\lambda}'(\eta)}{N_{1}(k)} \right\}. \tag{2.11}$ 

Here

$$\alpha_{0} = -\frac{\pi \left\{ C(y) + \frac{\gamma_{222}(y)}{\psi_{2}(k, \bar{h}) \gamma_{222}^{0}(y)} \left[ \frac{ik}{\pi} \bar{N}_{3}(k) C(y) + C_{2}(y) \bar{N}_{3}(k) \right] \right\}}{\bar{N}_{1}(k)} + \frac{\lambda(y)}{\psi_{1}(k)\pi} \gamma_{222}(y) \int_{-1}^{+1} \gamma_{221}(\xi) d\xi.$$

As follows from the solution to the plane problem of the nonstationary motion of a wing near a screen [7, 8], function  $\gamma(S)$  can be written as:

$$\gamma(S) = \psi_c a_0(y) \sqrt{\frac{1+S}{1-S}} + \psi \gamma_1^0(S) + \psi_a \gamma_2^0(S).$$

Functions  $\gamma_1^0(S)$ , which correspond to the plane problem for an unlimited fluid, have been calculated in [9]. By using these results, we get

$$a_{0} = \frac{a_{0}(y)}{2} - \frac{\pi}{N_{1}(k)} \left[ \frac{ik}{\pi} \overline{N}_{2}(k) C(y) + C_{2}(y) \overline{N}_{3}(k) \right] \left( \frac{\psi_{n}}{\psi_{2}} - 1 \right) + ik C_{3}(y) \frac{\psi_{n}}{\psi_{1}}. \quad (2.12)$$

where La' is the integral operator, whose core was obtained in monograph [4].

Now we must establish the connection between the functions which exclude  $\phi_1$  and  $\phi_2$  from (2.9). By definition when  $\lambda$   $\rightarrow$   $\infty$ 

$$a_{\lambda}(y) = \pi \left[ \frac{\psi_{c}a_{b}(y)}{2\lambda(y)} + \frac{\psi_{n}ik C_{b}(y)}{\lambda(y)} \right].$$

Constants  $C_1(y)$  take the form of

$$C_{2}(y) = \frac{2}{\pi} \int_{-1}^{+1} \frac{F_{1}(x)}{\sqrt{1-x^{2}}} dx = \frac{2}{\pi} \int_{-1}^{+1} \frac{[F_{1} - (N_{0} - N_{01})] A_{011}}{\sqrt{1-x^{2}}} dx_{0}^{2}$$

$$C_{3}(y) = -\frac{1}{\pi} \int_{-1}^{+1} \sqrt{\frac{1-x}{1+x}} F_{1}(x) dx.$$

From equation (2.9) we find

$$a_{\lambda}(y) = \frac{\pi\psi_{1}(k)}{\lambda(y)} a_{s}^{1};$$

$$a_{s}^{1} = a_{s} = \frac{a_{s}(y)}{2} + \frac{\psi_{n}}{\psi_{c}} ik C_{3}(y) - \frac{\pi}{\overline{N}_{1}(k)} \left[ \frac{ik}{\pi} \overline{N}_{3}(k) C(y) + C_{3}(y) \overline{N}_{3}(k) \right] \left( \frac{\psi_{n}}{\psi_{c}} - 1 \right).$$

By comparing the two expressions for  $\boldsymbol{a}_{\lambda}(\boldsymbol{y})\text{, we get the relationship we need$ 

$$\phi_n = \phi_2; \quad \phi_1 = \phi_0$$

Now, combining the results of equation (2.9), we write the intego-differential equation for a wing of great aspect ratio near a screen in a nonstationary flow

$$a_{\lambda}(y) = \frac{\pi \psi_{c}}{\lambda(y)} \left[ a_{s} - \frac{1}{2\pi |\overline{N}_{1}(k)|} \int_{-1}^{+1} a_{\lambda}'(\eta) K[k_{\lambda}(y - \eta)] d\eta, \qquad (2.13)$$

where

$$\alpha_0 = \frac{\alpha_0(y)}{2} + \frac{\psi_n}{\psi_c} ik C_2(y).$$

The core K (9) has the form of [4]

$$K(g) = \frac{N_{-1}P | y - \eta|}{(y - \eta)} + \frac{u - \eta}{(y - \eta) + 16\bar{h}^2} N_{-1} \left( P \sqrt{(y - \eta)^2 + 16\bar{h}^2} - \frac{4\bar{h}}{(y - \eta)^2 + 16\bar{h}^2} N_0 \left( P \sqrt{(y - \eta)^2 + 16\bar{h}^2} \right).$$

Where  $K \to 0$ ,  $N_1(k) \to -1$  and from (2.11) we get the Prandtl equation for a wing near a screen in a stationary flow [4]. Here

$$K(y-\eta) = \frac{1}{(y-\eta)} + \operatorname{sign} F \frac{(y-\eta)}{(y-\eta)^3 + 16\bar{h}^3}.$$

Thus, the Prandtl problem for a wing in a nonstationary flow is reduced to two one-dimensional integral equations (2.5), (2.13), where equation (2.5) corresponds to the equation of the stationary theory.

This result, which is the direct result of breaking the general solution down into three components, is significant in that the authors of known theory [1, 2], by introducing integral equations of the "reduced circulation" type, have found one one-dimensional equation.

3. In determining aerodynamic coefficients  $C_y$  and  $C_x$  a good first approximation gives us the solutions introduced in [3, 4].

$$\Gamma(y) = \Phi \sqrt{1 - \overline{y}^2}.$$

We use this representation for the approximate solution of (2.13). By assuming  $a_{\lambda}(y) = \phi \sqrt{1-y^2}$  for constant  $\phi$ , we get

$$\Phi = \frac{\frac{\pi \psi_e \alpha_s}{\lambda(0)}}{1 + \frac{\pi \psi_e}{2\lambda(0) + \overline{N}_k(k)}} \zeta k_{\lambda}, \qquad (3.1)$$

where  $\zeta k_{\lambda}$  is obtained from a formula in [4]:

$$U_{k_{\lambda}} = 1 - \frac{2}{\pi^{2}} \int_{-1}^{+1} \sqrt{1 - y^{2}} \int_{-1}^{+1} \frac{\eta}{\sqrt{1 - \eta^{2}}} \left\{ K\left[k_{\lambda}\left(y - \eta\right)\right] - \frac{1}{\left(y - \eta\right)} \right\} d\eta. \tag{3.2}$$

and  $k_{\lambda}$  - is the Strouhal span number.

The coefficient of the lifting force of the wing is calculated by formula

$$C_{\nu} = \frac{1}{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \gamma(\xi, \eta) d\xi d\eta. \qquad (3.3)$$

Now let us write the coefficient for the lifting force of a wing in the form of two components:

$$C_y = C_{y*} + C_{y*},$$

where  $C_{yB}$  is the coefficient of the lifting force determined by vortex movements;  $C_{yH}$  - the coefficient of the lifting force determined by the connected mass of the wing.

Now let us find components  $C_{yB}$  and  $C_{yH}$ :

$$C_{y^{0}} = -\frac{a_{\infty}\psi_{c}}{1 + \frac{a_{\infty}\psi_{c}}{\pi\lambda \lfloor \overline{N}_{1}(k) \rfloor}} \xi_{k_{\lambda}} \frac{C(k)}{\pi} \int_{-1}^{+1} \sqrt{\frac{1-x}{1+x}} F_{\bullet}^{\bullet}(x) dx +$$

$$+ 2\psi_{n} ik \int_{-1}^{+1} \sqrt{\frac{1-x}{1+x}} F_{\bullet}(x) dx \left[ 1 - \frac{1}{1 + \frac{a_{\infty}\psi_{c}}{\pi\lambda \lfloor \overline{N}_{1}(k) \rfloor}} \right] +$$

$$+ 2\psi \int_{-1}^{+1} \frac{F_{\bullet}(x)}{\sqrt{1-x^{3}}} x dx \left\{ 1 - \frac{\psi_{c}}{\psi \left[ 1 + \frac{a_{\infty}\psi_{c}}{\pi\lambda \lfloor \overline{N}_{1}(k) \rfloor} \zeta_{k_{\lambda}} \right]} \right\};$$

$$C_{y^{0}} = -\psi_{n} 2ik \int_{-1}^{+1} \sqrt{1-x^{3}} F_{1}(x) dx.$$

$$(3.4)$$

Here

$$F_{\bullet}(x) = F_{1}(x) - [N_{0} - N_{01}] \overline{A}_{271}$$

and C(k) is the Theodorsen coefficient [2].

In calculations the theoretical value of  $a_{\infty}$  =  $2\pi$  (with the effect of the viscosity of the fluid considered) can be assumed to be  $a_{\infty} \approx 5.45$ .

For a stationary flow  $C_{VH} = 0$ , while

$$C_{y_0} = -\frac{a_{\infty}\psi}{1 + \frac{a_{\infty}\psi}{\pi\lambda}} \int_{-1}^{+1} \sqrt{\frac{1-x}{1+x}} F_{s}(x) dx + 2\psi \int_{-1}^{+1} \frac{F_{s}(x)}{\sqrt{1-x^2}} x dx \left[1 - \frac{1}{1 + \frac{a_{\infty}\psi}{\pi\lambda} \zeta}\right].$$
 (3.6)

The first term in (3.6) gives us the well known value of the wing coefficient in a limited flow; the second term, which has an order of  $1/\lambda$ , represents the refinement which the present method introduces into the Prandtl theory. For a plane lifting surface the second term in (3.6) is equal to zero, and the results will be in complete agreement with the classical theory of the supporting line [3, 4].

Functions  $\phi_{\text{c}},~\phi_{\Pi},~\phi$  were obtained in [7, 8, 11] for forward and reciprocal wing oscillations.

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